# The Signifier as Aperture

# Toward an Optical Critique of Arbitrariness

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*The Real is not the void alone but the persistence of continuity through the void.*

*Before language communicates, it localizes (verortet).*

## Abstract

This article proposes that the camera obscura should be understood not merely as an optical instrument but as a materialization of a much older philosophical structure: *projection*. From Plato's cave and Neoplatonic theories of emanation to German Idealism and Lacanian psychoanalysis, appearance has repeatedly been conceived as the effect of a mediating operation through which presence becomes visible only by passing through absence. The camera obscura provides a privileged model of this operation because it transforms an external world into an inverted image through a minimal cut in space.

Building upon Lacan's Graph I, the present study argues that signification itself possesses a projective structure. The signifier is reconsidered not as a conventional label attached to a pre-given object but as an aperture through which objects, meanings, and subjects become projectable. This perspective allows a reconsideration of Saussure's doctrine of arbitrariness. While signifiers remain arbitrary with respect to resemblance, they are not arbitrary with respect to the conditions of appearance and localization (*Erscheinung* and *Verortung*). Like the aperture of the camera obscura, the signifier does not determine the identity of what appears, yet it participates decisively in the constitution of the field within which appearance becomes possible. The signifier does not resemble the world; it participates in the *Verortung des Erscheinens*: the localization of appearance.

The argument subsequently connects projection, the mirror stage, proper names, and Lacan's theory of the gaze to a broader topological account of continuity and division. Through a dialogue with Dedekind's construction of the real numbers, the article proposes that continuity is not the absence of gaps but their effect. The Real is approached not as a hidden substance behind language but as the impossible consistency generated through symbolic cuts. From this perspective, the camera obscura emerges not as a metaphor for signification but as one of its earliest material models. The Real itself is approached as a form of *Aus-stand*[1]: not a reality standing behind language, but a consistency that *stands out* of language precisely where language fails to coincide completely with itself.

[1] The term *Aus-stand* exploits a productive ambiguity in German. *Ausstand* ordinarily denotes a strike, stoppage, or interruption of work. Heard etymologically, however, it may also be understood as *Aus-stand*: a standing-out or standing-forth from within a structure. The Real is approached here in both senses simultaneously: as an internal interruption of the Symbolic and as what becomes manifest precisely through this interruption.

# Part I: Before the Camera Obscura or The World as Projection

The camera obscura was already there before the camera obscura.

Long before the seventeenth-century optical device transformed projection into a material apparatus, projection had already occupied a central place in philosophy, theology, mythology, and symbolic life. The device did not invent the structure; it rendered visible a problem that had accompanied human cultures for centuries.

From the perspective of ethnotopology[2], philosophy itself appears as only one manifestation of a broader projective structure. The forms through which a culture imagines the relation between appearance and reality, presence and absence, inside and outside, often reveal recurring topological operations that reappear in myths, dreams, rituals, architectures, and systems of thought alike.

Plato's cave already presents a theory of projection. The prisoners do not encounter things themselves but shadows cast upon a wall. Yet the cave is more than an epistemological allegory of illusion. It introduces a fundamental ontological problem: presence appears only through mediation.

Christianity radicalizes this structure through incarnation. The Word does not merely describe flesh; the Word becomes flesh. What was *projection* here assumes a sort of *embodiment*.

German Idealism transforms projection into dialectic. Thought and being, concept and object, Spirit and Nature become moments of a recursive process in which each emerges through the other.

Lacan inherits this lineage but reformulates it through signification. What German Idealism approached through dialectic, Lacan approaches through the signifier. Projection becomes linguistic, topological, and subjective at once.

The history of projection therefore does not culminate in the invention of the camera obscura. Rather, the camera obscura materializes a structure that philosophy had already been exploring and that cultures had long been inhabiting in myths, symbols, dreams, and ritual forms. The device was invented in the seventeenth century; the structure is much older. Plato's cave, Hegel's skull-bone, Persian/Arabic *ʿaks* عکس (reflection/inversion/image), the mirror stage, and Graph I all belong to the same genealogy of projection.[3]

# Lacan's Graph I and the Recursion of Projection

The most striking feature of Lacan's teaching is its recursive structure. From his reading of Descartes' *cogito ergo sum* to his own radical aphorisms—*"there is no woman"*, or *"a signifier is what represents the subject for another signifier"*—we encounter a logic that turns back upon itself. Lacan's discourse never proceeds linearly but instead folds, loops, and reflects, so that every

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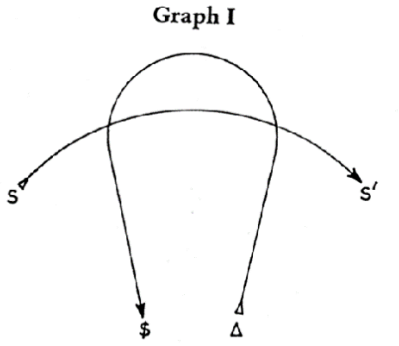
[2] Check Don. Kunze's essay: Ethnotopology, URL: https://bpb-us-e1.wpmucdn.com/sites.psu.edu/dist/9/19778/files/2024/08/ethnotopology.pdf ; last accessed on 5.6.2026.

[3] Esmaeilpour Ghoochani, Iraj: "Cinema Is a Film: In the Phenomenology of the Spirit of Iranian Cinema," accessed June 4, 2026,

https://www.academia.edu/98962139/Cinema_is_a_Film_In_the_phenomenology_of_the_spirit_of_Iranian_cinema?sm=a&rhid=40572078975.

proposition gestures beyond itself and back into the system that produced it. This recursive movement is the kernel of Lacanian thought.




This recursive kernel is nowhere more clearly represented than in **Graph I** from *"The Subversion of the Subject and the **Dialectic** of Desire in the Freudian Unconscious."* [4]The arc from **S → S'** captures the relational character of signification: meaning emerges only in the movement from one signifier to another. The second arrow stretched from **Δ** (the locus of the Other and of desire) toward **$** (the divided subject) shows that the subject never appears directly but only as split, inscribed by the signifying chain.

Graph I, then, is not a *closed map* but a loop in which subject and signifier are reciprocally constituted. It is a schema of recursion itself: a signifier only has meaning by referring to another, and the subject only emerges as represented within this passage. Now we will see that there is more into this as the diagram contains a further, often overlooked, possibility.

## The Topological Inversion

If we treat its arrows topologically—turning them inside-out—the symmetry of the graph comes to light. In this inversion, we no longer read: "a signifier represents the subject for another signifier", but rather: "***the subject relays the signifier to another subject, and is the very act of doing so***." This shift dramatizes the structure of speech. To speak is always to speak that could be to oneself and to the Other, and in doing so, both subject and signifier are altered.

Graph I can be reread as a recursive dynamical system in which subject and signifier continuously function as one another's input. The subject is not a stable element occupying a fixed position within the system but the trajectory generated through its recursive iteration: Speech produces a transformed subject; the transformed subject produces a transformed signifier; the transformed signifier produces another transformed subject, and so forth.

Neither subject nor signifier remains identical to itself from one iteration to the next. Each becomes the condition for the transformation of the other. What persists is therefore not a substantial identity but a recursive process of non-coincidence.

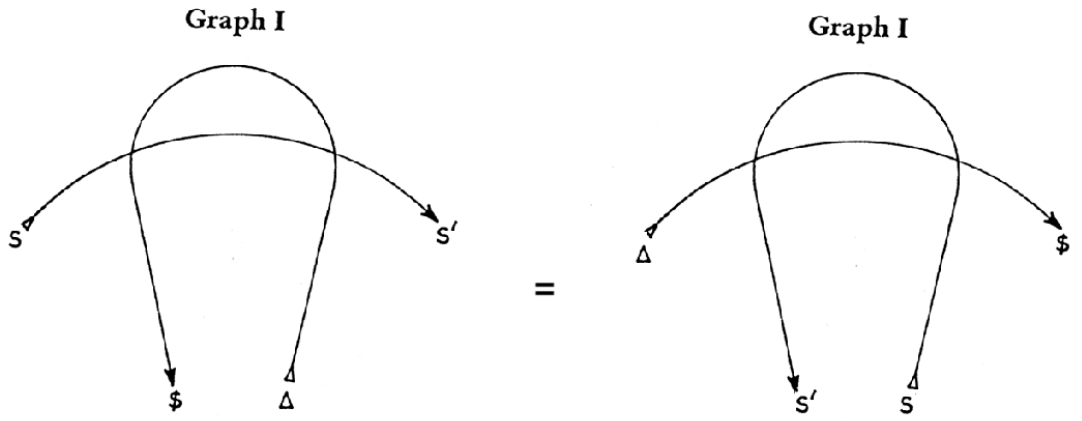
Then, Graph I resembles a dynamical system more than a static structure. The subject does not occupy a point within the graph; it traces a path through it. One might say that the subject is not an element contained within the structure, but a trajectory generated by its repeated operation. This recursive movement also clarifies why the subject can never fully coincide with itself which of course is a very Hegelean point. Every act of speech returns to the speaking subject transformed. The output of one iteration becomes the input of the next. Signification therefore

[4] *"The Subversion of the Subject and the Dialectic of Desire in the Freudian Unconscious"* is a lecture Lacan delivered at a colloquium of philosophers in Royaumont, France, from 19–23 September 1960, convened to explore the concept of the dialectic. The text engages not only with dialectical thought but also with "the subversion of the subject," a distinctly Hegelian notion. In Hegelian terms, the Master and Slave represent thesis and antithesis, whose interplay produces Identity as their synthesis—a dynamic Lacan interprets as the mechanics of subversion.

behaves less like a correspondence between fixed entities than like a process of continuous self-modification.

The signifier represents the subject for another signifier. Yet equally, the subject relays the signifier to another subject and becomes, in the very act of doing so, something other than it was. The graph therefore describes not a static relation but a circulation whose consistency depends upon perpetual transformation.

Such an inversion radicalizes Graph I: it reveals that the diagram always contained a hidden reciprocity, a symmetry between subject and signifier that mirrors the recursive logic of discourse:




**Inside-out of Graph I.** In Lacan's Graph I, the subject ($) appears as an effect of the passage from **S** to **S'**: *a signifier represents the subject for another signifier*. But if we treat the descending arrows as a continuous surface and turn it inside-out, the diagram licenses a complementary reading: ***the subject is what relays the signifier into the field of*** the ***Other***. Once touched by a signifier, the subject is divided; and in relaying that signifier to another subject, it is rewritten again. The two formulations are not opposites but two orientations of the same surface—each presupposes the other.

## Symmetry, Speech, and German Idealism: The Forgotten Symmetry of Graph I

Let us sum up all these: Lacan's Graph I is usually interpreted in only one direction: *a signifier represents the subject for another signifier*. Yet its very structure suggests a symmetry that has often been forgotten. If the arrows are inverted, the subject itself becomes the vector of the signifier. In this **structural reciprocity**, signifier and subject stand in the same order: **each only exists insofar as it passes through the other.**

This symmetry recalls Descartes' *cogito*, where thinking was long treated as the proof of being. Just as thought and being mirrored one another in Descartes, subject and signifier mirror one another in Lacan. In both cases, certainty emerges not from linear deduction but from recursive self-reference. Speech itself shows us this structure: when we speak, (nevermind its address is

ourselves and the Other subject,) and in the very act of speaking neither subject nor word remains the same.

This symmetry brings Lacan into direct dialogue with the tradition of German Idealism, where Descartes' problem of *thinking and being* was most radically reworked. Fichte, Schelling, and Hegel each proposed a doctrine of how concept and object, representation and being, stand to one another:

**Fichte's subjective idealism**: nothing possesses actuality apart from its relation to the activity of thought; reality is dependent on the self-positing "I."

**Schelling's objective idealism**: nature is accorded an independent status as precondition of the "I"; the subject is grounded in a reality external to it.

**Hegel's absolute idealism**: the antithesis between subjective and objective is resolved in the dialectical movement of Spirit, where both concept and being are continuously transformed.

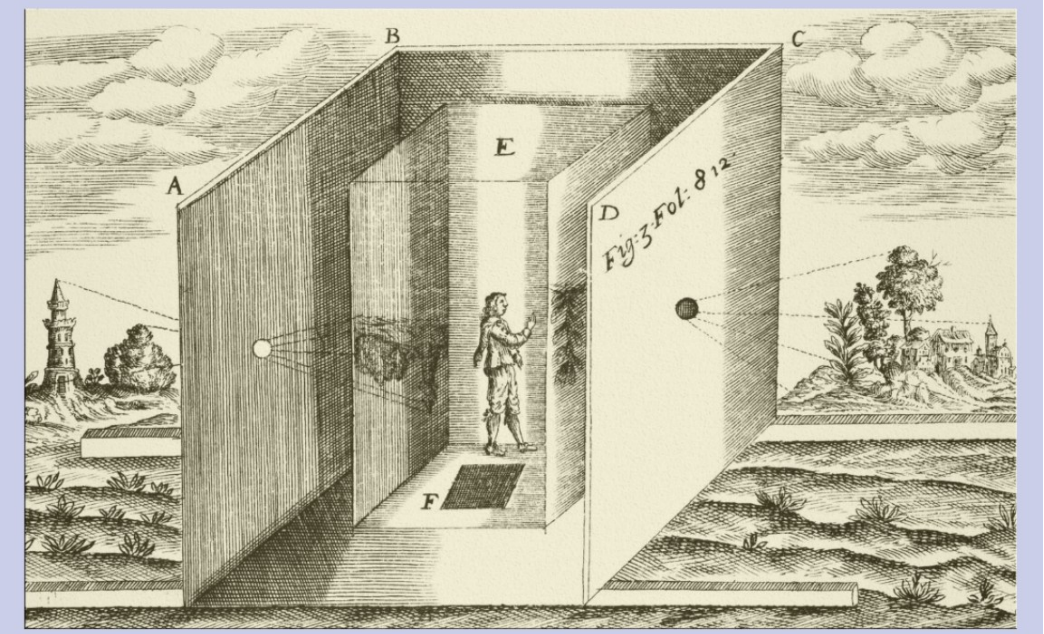
Seen through Lacan's topological lens, this lineage takes on new force. The very reciprocity of subject and signifier dramatizes the dialectical interplay of concept and being; Word and Flesh, Spirit and the Bone![5]

Thus, spirit is the skull and vice versa. After speech, not only is the subject altered, but the signifier itself becomes something different. The structure of discourse mirrors the structure of dialectic: both are recursive processes of becoming.

[5]Alluding to this sentence from Hegel: „*Die Wirklichkeit und Dasein des Menschen ist sein Schädelknochen!*" *The reality and Dasein of man is his skull-bone* One might follow the logic of this essay to interchange the relation between the Spirit and its image: The Bone! The original Hegelian text commonly referenced for this proposition reads: *"Die Wirklichkeit und Dasein des Menschen ist sein Schädelknochen... Wenn das Sein als solches oder Dingsein von dem Geiste prädiziert wird, so ist darum der wahrhafte Ausdruck hiervon, dass er ein solches wie ein Knochen ist."* [*The reality and concrete existence of man is his skull-bone... When Being as such or thingness is predicated of Spirit, the authentic expression of this is that Spirit is a thing like a bone*.] (Hegel, G.W.F., *Phänomenologie des Geistes*, Ullstein [1973], pp. 180-202)

For Hegel, the reconciliation of subjective and objective idealism occurs in the dialectical movement of Spirit. Spirit is not a static totality but a process of becoming, in which thought and being are mediated through their own negations. Lacan's Graph I, especially when read through its symmetrical inversion, provides a striking analogue of this movement. Yet the medium of transformation is different. In Hegel, concept and object transform one another through dialectical mediation. In Lacan, subject and signifier transform one another through speech, desire, and the circulation of jouissance. Every passage through the signifying chain leaves a remainder—objet petit a—which prevents closure and sets the circuit in motion again. The subject therefore persists not as a stable identity but as the trajectory generated through repeated encounters with its own constitutive non-coincidence. Both Hegel and Lacan think identity through movement, recursion, and negation; but where Hegel follows the labor of the concept, Lacan follows the detours of desire.

If subject and signifier emerge through projection, then projection is not merely a linguistic operation. We should expect to find homologous structures elsewhere. One such structure appears in the history of optics itself: the camera obscura.




Cámara Oscura descrpita en Ars Magna Lucis Et Umbrae 1646 – Athanasii Kircher 1601 – 1680

The camera obscura does not illustrate signification. It materializes a structure that philosophy had already discovered in Plato's cave, German Idealism, and Lacan's Graph I: every appearance emerges through projection, and every projection becomes the source of another projection.

Kircher’s engraving above is crucial because it does not merely show an optical apparatus. It shows an inhabitant inside the apparatus. The observer does not encounter the outside world directly; he encounters a projection. But this projection is not the end of the process. Once perceived, it becomes the source of another projection: interpretation, naming, judgment,

memory. The camera obscura therefore does not simply reproduce the world. It stages the passage from perception to signification. The decisive feature of Kircher's engraving is not the aperture but the observer. The projected image upon the wall is already a representation, yet for the observer it becomes the source of another operation: interpretation. The first projection transforms the external world into an image. The second transforms the image into meaning. The observer therefore inhabits a second camera obscura nested within the first. This recursive structure allows us to distinguish perception from conception. The image on the wall is not yet understanding. It must still be read. Yet this reading itself is another projection. Signification thus appears not as a transfer of information but as a chain of projections in which every image becomes the source of another image. The camera obscura therefore does not merely model vision. It models the recursive structure of signification itself.

Now, dear reader, something remarkable should become visible. Return for a moment to Lacan's Graph I. The machine before us is no longer merely an optical device. It is Graph I made wood, stone, and light. The world outside corresponds to the field from which signification emerges. The aperture performs the operation of selection and mediation. The projected image appears where meaning begins to take shape. Yet the true surprise stands quietly inside the chamber: the observer.

Without the observer, we have only optics.

With the observer, we have psychoanalysis.

For the projected image does not stop at the wall. It enters another circuit. It is perceived, interpreted, named, remembered, judged. The image becomes the source of another projection. The first camera obscura projects the world onto an image. The second projects the image into meaning.

Not exactly but in the same vain, speech cuts the subject, but in passing through the subject, the signifier itself is reconfigured. In other words, we can see the subject who cuts the signifier as a (stand-alone, quasi-automaton—an 'un-dead') subject, processing and relaying meaning. If not, "*what are poets for*?" (*Wozu Dichter*?') [6]

Inversion appears in more than one register. In optics it appears geometrically, as in the upside-down image of the camera obscura. In algebra it appears through reciprocal relations such as $y = 1/x$. In recursion it appears when a structure returns to itself only through transformation. The following examples should therefore not be read as mathematical curiosities but as different manifestations of the same operation: identity maintained through a passage into otherness.

## Inversion: The Hidden Logic of Projection

The camera obscura does more than project. It inverts. The image appearing on the interior wall is turned upside down. What was above appears below. What was left appears right. It presents the world through inversion. This detail is often treated as a merely optical consequence of the aperture. Yet inversion may be the deepest lesson of the camera obscura.

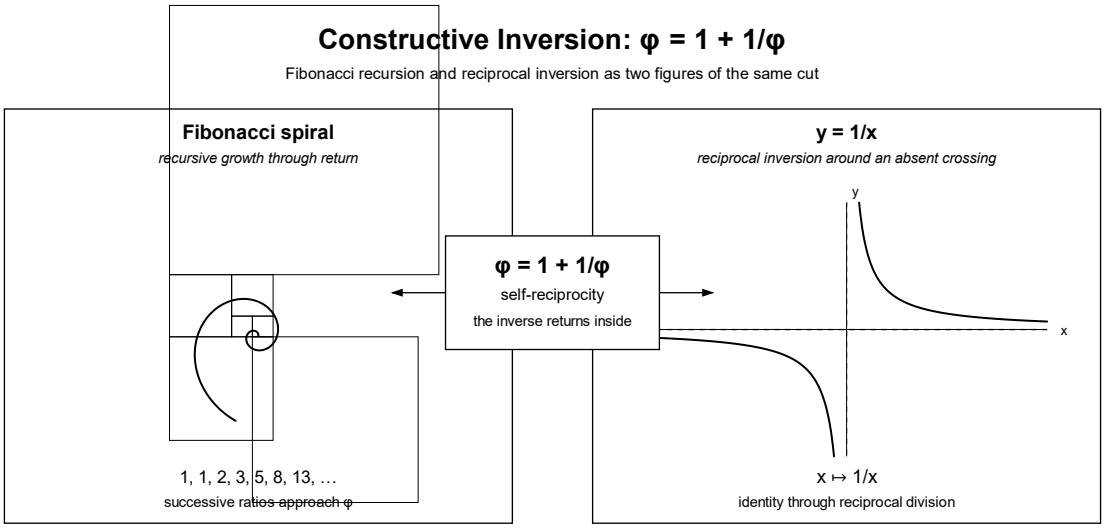
[6] In the age of AI, Heidegger's question—Wozu Dichter?—acquires renewed urgency. Machines can recursively transform signifiers, but poets dwell at the aperture where signification first becomes possible. If ordinary speech inhabits the projection, poetry remains closer to the cut through which language and being enter into relation. The poet does not merely manipulate signifiers but remains exposed to the event of signification itself.

For the history of philosophy repeatedly returns to structures that become intelligible only through reversal. To remember: Plato's cave inverts reality and appearance. Christianity inverts transcendence through incarnation. Hegel's dialectic proceeds through determinate negation and reversal (*Umkehrung*)[7].

The message returns from the Other in inverted form. Even the present study began by inverting Lacan's Graph I, revealing a hidden symmetry between subject and signifier.

The camera obscura therefore teaches a lesson that extends beyond optics. The Arabic and Persian word **ʿaks (عکس)** which literally stands for inversion, already preserves this intuition. An image is not simply a copy of the world but a reversal, a reflection, a turning-back. The photograph does not merely preserve reality; it produces a transformed relation to reality through inversion. From this perspective, inversion becomes a general principle of signification: The signifier is not the object; The image is not the source; The subject is not identical to itself, and so on…

Meaning emerges through a reversal that prevents any element from coinciding completely with its origin. Mathematics offers surprisingly similar examples. The hyperbola $y = 1/x$ is generated through reciprocal inversion. The two branches remain separated by asymptotes they can endlessly approach but never cross: Identity persists through division. Continuity is subjected to a fundamental cut. Likewise, the Fibonacci sequence generates growth through recursive return. Each term arises from a relation to preceding terms rather than from an independent origin. The sequence converges upon the golden ratio without ever arriving there. The limit remains constitutively absent.


Constructive Inversion: φ = 1 + 1/φ
Fibonacci recursion and reciprocal inversion as two figures of the same cut
Fibonacci spiral
recursive growth through return
y = 1/x
reciprocal inversion around an absent crossing
φ = 1 + 1/φ
self-reciprocity
the inverse returns inside
y
x
1, 1, 2, 3, 5, 8, 13, …
successive ratios approach φ
x ↦ 1/x
identity through reciprocal division

Both figures stage inversion as construction: the curve by reciprocal division, the spiral by recursive return toward φ.

In the golden ratio, $\phi=1+1/\phi$, the inverse is not external to the structure but returns inside it. This is the mathematical figure of what the article calls constructive inversion: continuity emerges not by excluding the cut, but by folding inversion into its own law. Lacan's use of the

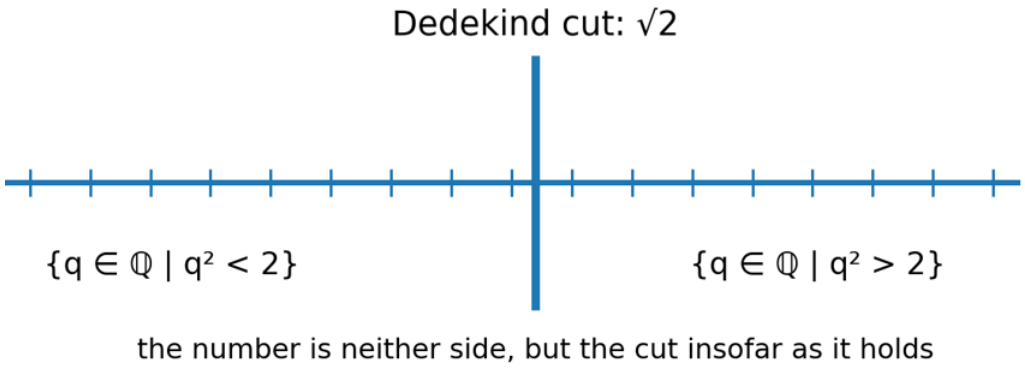
[7] Cf. Dieter Redlich, *Die Wende im Bild* (Bern: Peter Lang, 1999). Against interpretations that treat Marx's references to Hegelian "*Umkehrung*" (upside-down) as merely metaphorical, Redlich investigates inversion as a precise philosophical operation. In this respect, his work is of particular interest here, since the camera obscura, reflection (*ʿaks*), dialectical reversal, and Lacanian signification all depend upon forms of inversion that are constitutive rather than accidental.

Fibonacci series in Seminar XVI[8] similarly links recurrence, objet petit a, and the divided subject through an asymptotic structure that never closes into a perfect whole.

Lacan's subject behaves similarly: it includes its own inversion as an induction puzzle. The subject never encounters itself directly. It encounters itself only through signifiers returning from elsewhere, transformed by their passage through the Other. Every act of speech therefore contains a moment of inversion. The speaking subject returns to itself altered.

This recursive structure resembles the logic of induction puzzles. No single signifier produces a new subject, just as no single grain produces a heap[9], no single removed hair produces baldness, and no single term produces the Fibonacci limit. Yet with each iteration something shifts. The transformation cannot be localized positively within the sequence itself. One suddenly discovers oneself on the other side of a threshold that was never encountered as such.

The optical model of Camera Obscura, hence, becomes rigorous when placed beside Dedekind's construction of the real numbers. Dedekind's *Stetigkeit und irrationale Zahlen* [10](1872) does not treat irrational numbers as mysterious hidden points inserted into preexisting gaps. A real number is constructed through a cut, a *Schnitt* (*slot*), in the ordered field of rational numbers (Dedekind 1872).




Dedekind cut: the real number is the cut insofar as it holds continuity.

For example, $\sqrt{2}$ is not first a hidden point discovered somewhere between rationals. It is the cut separating $\{q \in \mathbb{Q} \mid q^2 < 2\}$ from $\{q \in \mathbb{Q} \mid q^2 > 2\}$. The number is therefore neither side nor an external filler, but the operation of division itself insofar as the division holds. This changes the ontology of continuity. Continuity is no longer what exists before division and later becomes fractured. Continuity emerges through division.

This was the missing mathematical core of the camera obscura argument. The aperture matters not because it contains the image but because it produces the conditions of projection. The Saussurean bar matters not because it transports a signified but because signification

[8] "Fibonacci Series," Learn Lacan, accessed June 6, 2026, https://www.learnlacan.com/concepts/fibonacci-series/.

[9] "(2) Réel," accessed June 6, 2026, https://www.academia.edu/168304897/R%C3%A9el.

[10] Richard Dedekind, *Stetigkeit und irrationale Zahlen*, with University of Michigan (Braunschweig : F. Vieweg, 1892), http://archive.org/details/stetigkeitundir00dedegoog.

emerges through its operation. Dedekind radicalizes the same structure: the cut does not destroy continuity; it constructs it.

At this point Lacan becomes legible in a new way. The Real is not empirical reality and not hidden fullness behind language. The Real emerges from the impossibilities or inconsistencies in the Symbolic, as Lacan repeatedly formulates in different ways in the late seminars. It is only through the Symbolic that one is introduced to the Real, because without an inside there is no outside. The Real stands out of language in a kind of *Aus-stand*: it stands outside, but also marks the point where functioning stops. The Real appears where the Symbolic fails to coincide fully with itself, yet nevertheless continues to hold.

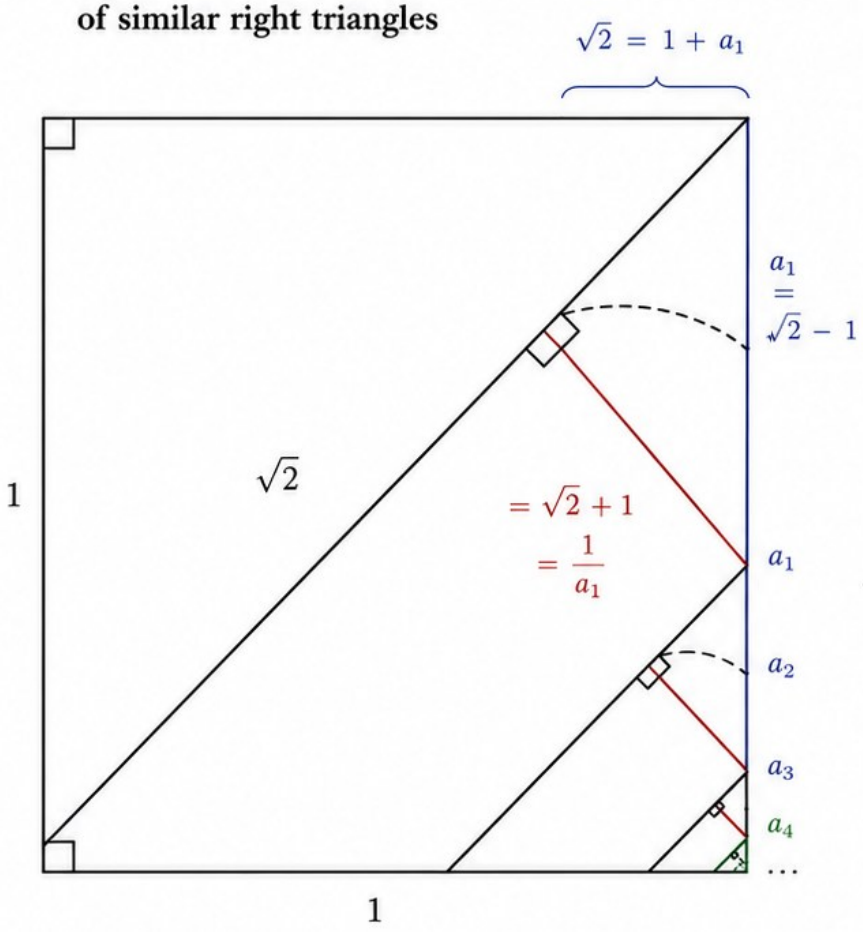
Thus and again, the Real is not the void alone, but the persistence of continuity through the void. The Symbolic stitches this impossible continuity together; the Imaginary experiences it as coherent form. As we will see, this triadic formulation will later return to the mirror stage and the proper name.

Lacan's appeal to the Fibonacci sequence may be understood as a special case of a more general principle. The essential feature is not the Fibonacci numbers themselves but the persistence of identity through recursive non-coincidence. A structure returns to itself only by passing through an operation that transforms it.

Dedekind's irrational numbers reveal the same logic. The number is never encountered directly but emerges through an endless refinement of a constitutive cut. Identity is not located in any particular approximation but in the persistence of the partition through successive transformations. The subject behaves similarly. It is not a positive element hidden behind signifiers but the consistency generated by their recursive passage. The subject persists because it never fully coincides with itself.

What Fibonacci recursion, continued fractions, Dedekind cuts, and Lacanian subjectivity share is therefore not a particular numerical formula but a common topology: identity generated through recursive division.

# $\sqrt{2}$ from recursive construction of similar right triangles




## 1. First step

In the unit square, the diagonal has length $\sqrt{2}$.

Subtract 1 from $\sqrt{2}$:

$$a_1 = \sqrt{2} - 1 \approx 0.41421356\ldots$$

Thus $\sqrt{2} = 1 + a_1$.

Invert $a_1$:

$$\frac{1}{a_1} = \sqrt{2} + 1 \approx 2.41421356\ldots$$

The number returns, transformed.

Relation:

$$a_1 = \sqrt{2} - 1 = \frac{1}{\sqrt{2} + 1}$$

## 2. Recursive construction

Use $a_1$ as the leg on the right side to build the next similar right triangle on the diagonal.

This gives the next remainder:

$$a_2 = \frac{1}{\frac{1}{a_1} - 2} = \frac{a_1}{1 + a_1} \approx 0.17157288\ldots$$

Invert:

$$\frac{1}{a_2} = 2 + a_1 \approx 2.17157288\ldots$$

Repeat indefinitely:

$$a_{n+1} = \frac{a_n}{1 + a_n} \qquad (n \geq 1)$$

Each new triangle is similar to the previous one.
The remainders $a_n$ decrease to 0.

## 3. Fixed-point (self-reciprocity)

The number $a = \sqrt{2} - 1$ is a fixed point of the map $a \mapsto \frac{a}{1+a}$:

$$a = \frac{a}{1 + a} \iff a^2 + a - a = 0 \iff a^2 + 2a - 1 = 0$$
$$\iff a = \sqrt{2} - 1.$$

Equivalently:

$$a = \frac{1}{2 + a} \iff \sqrt{2} - 1 = \frac{1}{2 + (\sqrt{2} - 1)}$$

## 4. Connection with the golden ratio $\varphi$

The golden ratio satisfies

$$\varphi = 1 + \frac{1}{\varphi}$$

Our number satisfies

$$\sqrt{2} - 1 = \frac{1}{2 + (\sqrt{2} - 1)}$$

Both are self-referential (fixed-point) relations of the form

$$x = c + \frac{1}{x} \qquad \text{(for a constant } c\text{)}$$

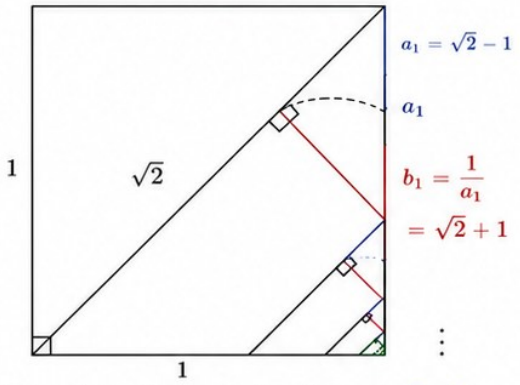
| Number | Equation | Constant $c$ | Value |
|---|---|---|---|
| $\varphi$ | $x = 1 + \frac{1}{x}$ | 1 | 1.6180339... |
| $\sqrt{2} - 1$ | $x = 2 + \frac{1}{x}$ | 2 | 0.41421356... |

The same recursive inversion structure underlies both $\sqrt{2}$ and $\varphi$.
It is the algebraic signature of their deep resemblance.

# Recursive Inversion: A General Principle for Irrational Numbers

*Not every irrational number satisfies the same fixed equation.*
*What is general is the procedure: by subtraction, inversion, and renewed approximation, every irrational number is approached through an infinite recursive process. In special cases the pattern stabilizes; in general it does not.*

## A. Example 1: $\sqrt{2}$ (a quadratic irrational)




1. Start with the unit square. The diagonal has length $\sqrt{2} = 1 + a_1$.
2. Subtract 1: $a_1 = \sqrt{2} - 1$.
3. Invert: $b_1 = \frac{1}{a_1} = \sqrt{2} + 1$.
4. Use $b_1$ to build the next right triangle on the diagonal.
5. Repeat indefinitely.

**The recursion stabilizes with a constant pattern**

$a_{n+1} = \frac{1}{2 + a_n}$ $(a_0 = 1)$

Fixed point: $a = \frac{1}{2+a} = \sqrt{2} - 1$

Thus $\sqrt{2} = 1 + a$.

Continued fraction (periodic):

$$\sqrt{2} = [1;\overline{2}] = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cdots}}}$$

Special feature: the coefficients repeat (period 1).
This is typical of quadratic irrationals.

## B. General principle (valid for every irrational number)

Every irrational number $\alpha$ has a unique continued fraction expansion

$$\alpha = a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + + \cdots}}}$$

$a_0 \in \mathbb{Z}, \quad a_k \in \mathbb{N} \quad (k \geq 1)$

This is equivalent to the recursive process

$$r_0 = \alpha, \quad a_0 = \lfloor r_0 \rfloor, \quad r_1 = \frac{1}{r_0 - a_0}$$

$$r_{n+1} = \frac{1}{r_n - a_n}, \quad a_n = \lfloor r_n \rfloor \quad (n \geq 1)$$

**What is general:**

- At each step we subtract an integer part $a_n$ (a "cut").
- We invert the remainder $r_n$ (a "return").
- We iterate indefinitely.
- Rational numbers terminate (the remainder becomes 0).
- Irrational numbers never terminate.

| Examples | $\sqrt{2}$ $=[1;\overline{2}]$ | $\sqrt{3}$ $=[1;\overline{1,2}]$ | $\phi = \frac{1+\sqrt{5}}{2}$ $=[1;\overline{1}]$ | $\pi$ $=[3;7,15,1,292,\ldots]$ |
|---|---|---|---|---|
| $a_0$ | 1 | 1 | 1 | 3 |
| $a_1$ | 2 | 1 | 1 | 7 |
| $a_2$ | 2 | 2 | 1 | 15 |
| $a_3$ | 2 | 1 | 1 | 1 |
| $a_4$ | 2 | 2 | 1 | 292 |
| $a_5$ | 2 | 1 | 1 | 1 |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |
| | Periodic continued fractions ( patterns stabilize ) | | | Non-periodic continued fraction (no simple repeating pattern) |

Dedekind allows us to generalize Lacan's Fibonacci intuition into a theory of identity generated through recursive division.

The subject is constituted precisely through this impossible passage. Like the irrational number emerging from a Dedekind cut, it appears not as an element contained within the chain but as the consistency generated by its recursive division. The subject therefore does not merely undergo inversion; it is the persistence of identity through inversion. Every return is a return as other.The camera obscura makes this visible.

Projection is inversion.

Signification is inversion.

Subjectivity is inversion.

The world appears only after passing through a cut that turns it inside out.

The recursive inversion visible in √2 and φ should not be mistaken for a special property of particular numbers. It is the visible manifestation of a more general topological operation. In quadratic irrationals, the operation appears algebraically through self-similar recursive equations. In circles, however, it appears geometrically.

The examples of √2 and φ may suggest that recursive inversion belongs only to certain mathematical structures. Aristotle's wheel reveals a more general possibility. Here inversion no

longer appears algebraically but topologically. The paradox emerges because difference is folded into continuity. What Dedekind localizes as a cut, Aristotle distributes as a fold.

Aristotle's wheel reveals this hidden structure.11 The paradox emerges because unequal circumferences are forced to advance as one. Difference is not eliminated but folded into continuity. As the circle is approximated by polygons, the discontinuities do not vanish. They lose their location and become distributed throughout the motion. The gap persists as a non-localizable remainder. The same logic underlies Dedekind's construction of irrational numbers. The irrational is never reached directly; it persists as the remainder generated by an endless process of localization.

The wheel therefore provides for circles what the Dedekind cut provides for irrational numbers. Both reveal that continuity is not primary. Continuity is an effect produced by a structured gap. In the Dedekind cut, the gap is progressively localized. In Aristotle's wheel, it is progressively delocalized. In both cases, identity persists only through a fold that prevents complete coincidence.

Why does this matter for language? Because the signifier functions in an analogous manner. It neither transmits a pre-existing identity nor creates one ex nihilo. Like the aperture, the cut, and the fold, it generates continuity through division.

The genius of Dedekind's cut, Saussure's bar, and Lacan's barred subject lies in a common discovery: the void and the division are not obstacles to continuity but its very condition.

At first glance, a cut appears as a lack. It separates, interrupts, and prevents coincidence. Yet Dedekind showed that continuity itself is generated through a cut. The irrational number does not fill the gap; the gap holds. Likewise, Saussure's bar separating signifier and signified is not a wall standing between two already constituted entities. It is the very operation through which signification becomes possible.

Lacan radicalizes this insight. The barred subject ($) is not a complete subject subsequently wounded by division. The bar is constitutive. Subjectivity emerges through the impossibility of coinciding fully with itself.

The remarkable consequence is that the bar and the void cease to be opposites. The bar gives the void a structure; the void gives the bar its efficacy. Dedekind's cut, Saussure's bar, the aperture of the camera obscura, and the Lacanian subject all reveal the same paradoxical principle: continuity is constructed through division.

What appears as absence is an operation. What appears as interruption is a condition of persistence. The cut does not threaten continuity. The cut is what allows continuity to hold.

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[11] Cf. one of my former essays: “The Gap as Fold Aristotle’s Wheel and the Temporal Twist of Space,” accessed June 6, 2026, https://www.academia.edu/166193540/The_Gap_as_Fold_Aristotles_Wheel_and_the_Temporal_Twist_of_Space.